Modal Logic Reasoning: The Long View

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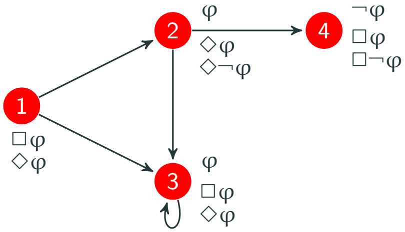
Table of contents

- Modal logics are among the most widely and extensively studied logical formalism
- We will focus on the so-called modal cube, extensions of basic modal logic with arbitrary subsets of five additional axioms
- We will consider a small number of approaches to reasoning in the logics of the modal cube that have been proposed over several decades
  - How effective are they?
  - What benchmarks can we use for this purpose?
  - Did we make progress?

- Basic modal logic K: Propositional logic extended with unary operators
  $\Box$ ('box', 'necessarily') and
  $\Diamond$ ('diamond', 'possibly')
- Semantics of modal formulae is given by Kripke structures
  $M = \langle\langle W, R\rangle, V\rangle = \langle W, R, V\rangle$ where
  – W is a set of world
  – R is a binary accessibility relation on W
  – V is a valuation: $V(p) \subseteq W$ for $p \in P$



$M, w \models p$ iff $w \in V(p)$

$M, w \models \neg\varphi$ iff $M, w \not\models \varphi$

$M, w \models \varphi \vee \psi$ iff $M, w \models \varphi$ or $M, w \models \psi$

$M, w \models \Box\varphi$ iff for every $v \in W$, if wRv then $M, v \models \varphi$

$M, w \models \Diamond\varphi$ iff for some $v \in W$, wRv and $M, v \models \varphi$

- Basic logic K: Propositional logic extended with unary operators
  $\Box$ ('box', 'necessarily') and
  $\Diamond$ ('diamond', 'possibly')
- Semantics of modal formulae is given by Kripke structures
  $M = \langle\langle W, R\rangle, V\rangle = \langle W, R, V\rangle$ where
  - W is a set of world
  - R is a binary accessibility relation on W
  - V is a valuation: $V(p) \subseteq W$ for $p \in P$

A formula φ is K-satisfiable iff there is a (finite tree) Kripke structure M with root $w \in W$ such that $\langle M, w\rangle \models \varphi$

PSPACE-complete problem [Ladner 77, Halpern and Moses 92]

- In a tree Kripke structure M every world $w \in W$ has a unique modal level, $\mathrm{ml}_M(w)$, given by the distance of w to the root

Proof System hK for Modal Logic K =

Axioms of Propositional Logic

+ K / Normality

$\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$

+ Duality

$\Box\varphi \equiv \neg\Diamond\neg\varphi$

+ Modus Ponens

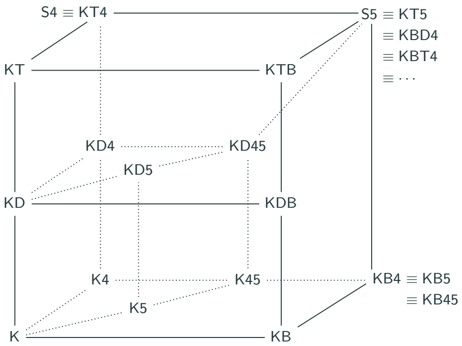
$(\varphi \rightarrow \psi), \varphi \vdash \psi$

+ Necessitation

$\varphi \vdash \Box\varphi$

A formula φ is provable in hK iff $\neg\varphi$ is not K-satisfiable

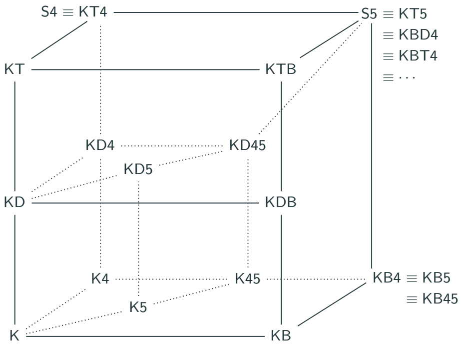
	Axiom	Frame Property
D	$\Box\varphi \rightarrow \Diamond\varphi$	Serial $\forall v \exists w . vRw$
T	$\Box\varphi \rightarrow \varphi$	Reflexive $\forall w . wRw$
B	$\varphi \rightarrow \Box\Diamond\varphi$	Symmetric $\forall vw . vRw \rightarrow wRv$
4	$\Box\varphi \rightarrow \Box\Box\varphi$	Transitive $\forall uvw . (uRv \wedge vRw) \rightarrow uRw$
5	$\Diamond\varphi \rightarrow \Box\Diamond\varphi$	Euclidean $\forall uvw . (uRv \wedge uRw) \rightarrow vRw$



There are 15 distinct logics that can be formed by adding a subset of $\{B, D, T, 4, 5\}$ to hK

A formula φ is KΣ-provable iff it is provable in hK + Σ.

	Axiom	Frame Property
D	$\Box\varphi \to \Diamond\varphi$	Serial $\forall v \exists w. vRw$
T	$\Box\varphi \to \varphi$	Reflexive $\forall w. wRw$
B	$\varphi \to \Box\Diamond\varphi$	Symmetric $\forall vw. vRw \to wRv$
4	$\Box\varphi \to \Box\Box\varphi$	Transitive $\forall uvw. (uRv \land vRw) \to uRw$
5	$\Diamond\varphi \to \Box\Diamond\varphi$	Euclidean $\forall uvw. (uRv \land uRw) \to vRw$



There are 15 distinct logics that can be formed by adding a subset of $\{B, D, T, 4, 5\}$ to hK

A formula φ is KΣ-satisfiable iff there is a (rooted) Kripke structure $M = \langle W, R, V \rangle$ and $w \in W$ such that (1) R satisfies all the frame properties corresponding to axioms in Σ
(2) $\langle M, w \rangle \models \varphi$
M is a KΣ-model of φ

The Modal Cube

	Axiom	Frame Property
D	$\Box\varphi \to \Diamond\varphi$	Serial $\forall v \exists w . vRw$
T	$\Box\varphi \to \varphi$	Reflexive $\forall w . wRw$
B	$\varphi \to \Box\Diamond\varphi$	Symmetric $\forall vw . vRw \to wRv$
4	$\Box\varphi \to \Box\Box\varphi$	Transitive $\forall uvw . (uRv \land vRw) \to uRw$
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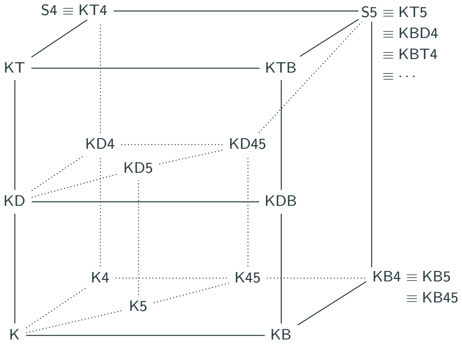


There are 15 distinct logics that can be formed by adding a subset of $\{B, D, T, 4, 5\}$ to hK

A formula φ is is $K\Sigma$-provable iff $\neg\varphi$ is not $K\Sigma$-satisfiable

- We want to determine whether a modal formula φ is KΣ-satisfiable / KΣ-provable
- Idea 1: We can simply write down the semantics of φ as first-order formula and give it to a first-order theorem prover

$M, w \models p$ iff $w \in V(p)$

$M, w \models \Box\varphi$ iff for every $v \in W$, if wRv then $M, v \models \varphi$

$M, w \models \varphi \vee \psi$ iff $M, w \models \varphi$ or $M, w \models \psi$

$M, w \models \Diamond\varphi$ iff for some $v \in W$, wRv and $M, v \models \varphi$

$\leadsto$ we do not need to include M in the formula

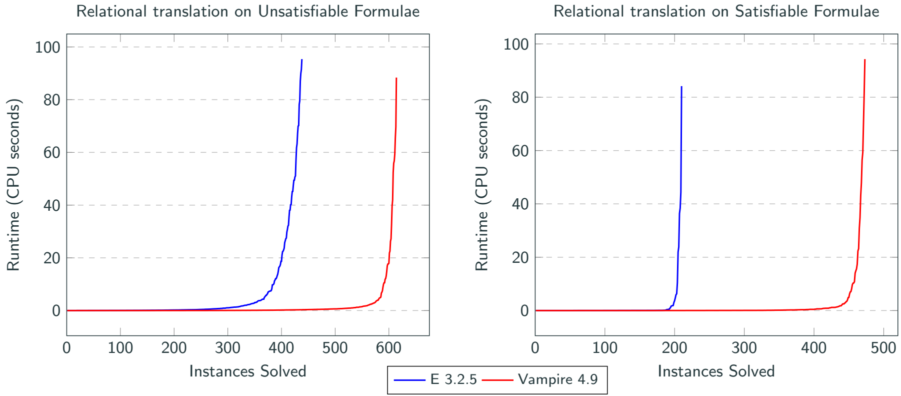
$\leadsto$ instead of $M, w \models p$ we can then write $p(w)$

$\Box p \wedge \Diamond\neg p$	becomes	$\forall v.R(w,v) \rightarrow p(v) \wedge \exists v.R(w,v) \wedge \neg p(v)$
$\Box p \wedge \Box\neg p$	becomes	$\forall v.R(w,v) \rightarrow p(v) \wedge \forall v.R(w,v) \rightarrow \neg p(v)$

- To deal with Σ we add the corresponding frame properties to the first-order formula

How well does Idea 1 work?

- For benchmarking we use 100 satisfiable and 100 unsatisfiable modal formulae for each logic
  ⇝ 1500 satisfiable and 1500 unsatisfiable formulae in total
- Each prover is given 100 CPU seconds to solve each formula
  (median time over five runs)



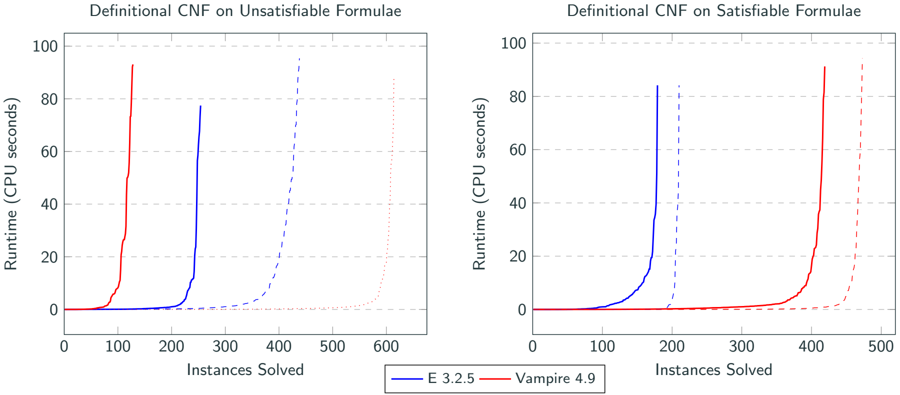
How well does Idea 1 work?

- For each logic Rel(KΣ), the Rel(KΣ) subfragment of FOL consisting of formulae obtained as the relational translation of modal formulae for KΣ has a decidable satisfiability problem
- But standard first-order calculi (resolution) are not decision procedures for any of the Rel(KΣ) fragments

- Idea 2:
  Introduce new names for each non-atomic subformula of a modal formula φ / the corresponding subformulae of its relational translation
- Possible clauses:

$$\begin{array}{ll} \neg q_{\neg p}(w) \vee \neg p(w) & \neg q_{\varphi\vee\psi}(w) \vee q_{\varphi}(w) \vee q_{\psi}(w) \\ \neg q_{\Box\varphi}(w) \vee \neg R(w,v) \vee q_{\varphi}(v) & \neg q_{\Diamond\varphi}(w) \vee R(w, f_{\Diamond\varphi}(w)) \\ & \neg q_{\Diamond\varphi}(w) \vee q_{\varphi}(f_{\Diamond\varphi}(w)) \end{array}$$

- We deal with Σ again by adding the corresponding frame properties as before
- We can then obtain decision procedures based on ordered resolution for all $K\Sigma$ where $4, 5 \notin \Sigma$


Definitional CNF on Unsatisfiable Formulae
Runtime (CPU seconds)
Instances Solved
0
20
40
60
80
100
0
100
200
300
400
500
600
Definitional CNF on Satisfiable Formulae
Runtime (CPU seconds)
Instances Solved
0
20
40
60
80
100
0
100
200
300
400
500
E 3.2.5
Vampire 4.9

How well does Idea 2 work?

- Problem 1: Resolution steps neither respect modal levels nor the distinction between $\Diamond$ and $\Box$
- Problem 2: Resolution steps are 'too small'

Consider $\Diamond((p \vee q) \wedge \Diamond(\neg p \vee q)) \wedge \Box\neg q$

(1)	$q_1(w_0)$	[R,(6)2,(9)2]	(12)	$\neg q_3(V) \vee \neg q_5(V) \vee q(V)$
(2)	$\neg q_1(V) \vee R(V, f(V))$	[R,(7)2,(10)2]	(13)	$\neg q_1(V) \vee \neg q_4(V) \vee q_6(g(V))$
(3)	$\neg q_1(V) \vee q_2(f(V))$	[R,(9)3,(11)2]	(14)	$\neg q_5(V) \vee \neg p(V) \vee \neg q_6(V)$
(4)	$\neg q_2(V) \vee q_3(V)$	[R,(13)3,(14)3]	(15)	$\neg q_1(V) \vee \neg q_4(V) \vee \neg q_5(g(V)) \vee \neg p(g(V))$
(5)	$\neg q_2(V) \vee q_4(V)$			
(6)	$\neg q_3(V) \vee p(V) \vee q(V)$	[R,(8)2,(15)3]	(16)	$\neg q_1(V) \vee \neg q_4(V) \vee \neg p(g(V))$
(7)	$\neg q_4(V) \vee R(V, g(V))$		$\vdots$	
(8)	$\neg q_4(V) \vee q_5(g(V))$			
(9)	$\neg q_5(V) \vee \neg p(V) \vee q(V)$			
(10)	$\neg q_1(V) \vee \neg R(V, W) \vee q_6(W)$			
(11)	$\neg q_6(V) \vee \neg q(V)$			

- Idea 3: Use a modal clausal normal form and modal resolution calculus
  To deal with axioms in Σ, add instances of the axioms
- Recall that every K-satisfiable formula has a rooted tree Kripke model
  where every world w has a unique modal level, $\mathrm{ml}_M(w)$ given by the distance of w to the root
- Separated Normal Form with Sets of Modal Levels SNF_{sml}
  - Literal clause $S : \bigvee_{i=1}^{k} l_i$ — $\forall w.$ if $\mathrm{ml}_M(w) \in S$ then $M, w \models \bigvee_{i=1}^{k} l_i$
  - Positive modal clause $S : l' \rightarrow \Box l$ — $\forall w.$ if $\mathrm{ml}_M(w) \in S$ then $M, w \models l' \rightarrow \Box l$
  - Negative modal clause $S : l' \rightarrow \Diamond l$ — $\forall w.$ if $\mathrm{ml}_M(w) \in S$ then $M, w \models l' \rightarrow \Diamond l$

  where $S \subseteq \mathbb{N}$ (possibly infinite)
- Disjunctions here are are sets of literals (not multi-sets)

Only $0 : \bot$, where $\bot$ is the empty disjunction of literals, is a clause that is K-unsatisfiable on its own

- The transformation to normal form ρ_L starts with

$$\{\ \{0\} : t_\varphi, \quad \{0\} : t_\varphi \to \varphi\ \}$$

and uses a new propositional symbol (surrogate) $\eta(\psi) = t_\psi$ for (almost) every subformula ψ of φ

KD ($\Box\psi \to \Diamond\psi$):

$$\Phi \cup \{S : t_{\Box\psi} \to \Box\eta(\psi)\} \quad \Longrightarrow \quad \Phi \cup \{S : t_{\Box\psi} \to \Box\eta(\psi), S : t_{\Box\psi} \to \Diamond\eta(\psi), S^{\geqslant} : \eta(\psi) \to \psi\}$$

KT ($\Box\psi \to \psi$):

$$\Phi \cup \{S : t_{\Box\psi} \to \Box\eta(\psi)\} \quad \Longrightarrow \quad \Phi \cup \{S : t_{\Box\psi} \to \Box\eta(\psi), S : \neg t_{\Box\psi} \vee \eta(\psi) S \cup S^{+} : \eta(\psi) \to \psi\}$$

K4 ($\Box\psi \to \Box\Box\psi$):

$$\Phi \cup \{S : t_{\Box\psi} \to \Box\eta(\psi)\} \quad \Longrightarrow \quad \Phi \cup \{S^{\geqslant} : t_{\Box\psi} \to \Box\eta(\psi), S^{\geqslant} : t_{\Box\psi} \to \Box t_{\Box\psi}, (S^{+})^{\geqslant} : \eta(\psi) \to \psi\}$$

where $S^{+} = \{l + 1 \in \mathbb{N} \mid l \in S\}$, $S^{\geqslant} = \{n \mid n \geqslant \min(S)\}$,

Theorem

Let L be a logic in the modal cube,
φ *be a modal formula and*
$\Phi = \rho_L(\varphi)$ *be the normal form of* φ *for L.*

Then φ *is L-satisfiable iff* Φ *is K-satisfiable*

$$\text{LRES}: \frac{\begin{array}{rl} S_0: & D \vee l \\ S_1: & D' \vee \neg l \end{array}}{S_0 \cap S_1 : D \vee D'} \qquad \text{MRES}: \frac{\begin{array}{rl} S_0: & l_1 \rightarrow \Box l \\ S_1: & l_2 \rightarrow \Diamond \neg l \end{array}}{S_0 \cap S_1 : \neg l_1 \vee \neg l_2} \qquad \text{GEN2}: \frac{\begin{array}{rl} S_0: & l'_1 \rightarrow \Box l_1 \\ S_1: & l'_2 \rightarrow \Box \neg l_1 \\ S_2: & l'_3 \rightarrow \Diamond l_2 \end{array}}{\bigcap\{S_0, S_1, S_2\} : \neg l'_1 \vee \neg l'_2 \vee \neg l'_3}$$

$$\text{GEN1}: \frac{\begin{array}{rlcl} S_0: & l'_1 & \rightarrow & \Box\neg l_1 \\ & \vdots & & \\ S_{m-1}: & l'_m & \rightarrow & \Box\neg l_m \\ S_m: & l' & \rightarrow & \Diamond\neg l \\ S_{m+1}: & \multicolumn{3}{l}{l_1 \vee \cdots \vee l_m \vee l} \end{array}}{S: \quad \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'} \qquad \text{GEN3}: \frac{\begin{array}{rlcl} S_0: & l'_1 & \rightarrow & \Box\neg l_1 \\ & \vdots & & \\ S_{m-1}: & l'_m & \rightarrow & \Box\neg l_m \\ S_m: & l' & \rightarrow & \Diamond l \\ S_{m+1}: & \multicolumn{3}{l}{l_1 \vee \cdots \vee l_m} \end{array}}{S: \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'}$$

where $S = \bigcap\{S_0, \ldots, S_m, S^-_{m+1}\}$ in GEN1 and GEN3

$S^-_{m+1} = \{ml - 1 \in \mathbb{N} \mid ml \in S_{m+1}\}$

and inference steps are only performed if the labelling set in the resolvent is non-empty

- Tautology:

  $S : p \vee \neg p \vee C$ is a tautology

- Subsumption:

  $S_0 : C$ subsumes $S_1 : C \vee D$ where $S_1 \subseteq S_0$ and D is a possibly empty disjunction of literals

- Derivation from Φ:

  $\Phi = \Phi_0, \Phi_1, \ldots,$ where for each $i > 0$,

  (i) $\Phi_{i+1} = \Phi_i \cup \{ml : C\}$ where $ml : C$ is derived from clauses in Φ_i,
  not a tautology,
  and not subsumed by a clause in Φ_i; or

  (ii) $\Phi_{i+1} = \Phi_i - \{ml : C\}$ where $ml : C$ is subsumed by a clause in $\Phi_i - \{ml : C\}$

- Refutation of Φ:

  A derivation $\Phi = \Phi_0, \Phi_1, \ldots, \Phi_n$ where Φ_n contains $S : \bot$ with $0 \in S$

- Φ is saturated:

  no clause can be derived from Φ that is not a tautology or subsumed by a clause in Φ

A Calculus for SNF_{sml}

- Efficiency of the calculus can be improved by restricting the applicability of the LRES ('propositional' binary resolution) rule
  - Negative resolution: One of the premises contains only negative literals

    Additional restrictions on normal form:

    Only positive literals in modal clauses $\rightsquigarrow$ SNF^{-}_{sml}
  - Positive resolution: One of the premises contains only positive literals

    Additional restrictions on normal form:

    Only negative literals in modal clauses $\rightsquigarrow$ SNF^{+}_{sml}
  - Ordered resolution: For premises $C \vee l$ and $D \vee \neg l$, l must be maximal wrt to C and $\neg l$ must be maximal wrt to D

    Additional restrictions on normal form and ordering:

    Below modal operators we must have fresh propositional symbols that are smaller than the original propositional symbols $\rightsquigarrow$ SNF^{++}_{sml}
- The additional restrictions on the normal form can be enforced by additional renamings

Let

L be a logic in the modal cube

φ be a modal formula

Φ be the corresponding finite set of clauses in SNF_{sml}, SNF^{-}_{sml}, SNF^{+}_{sml}, or SNF^{++}_{sml}

Φ' be the saturation of Φ with respect to the corresponding refinement of the calculus for SNF_{sml}

Theorem

φ is L-unsatisfiable iff Φ has a refutation with respect to the corresponding refinement of the calculus

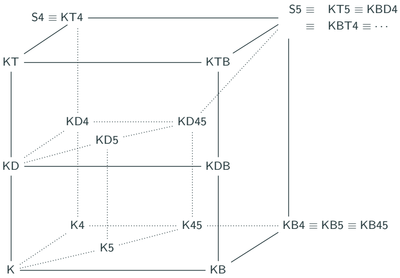
Theorem

If φ is L-satisfiable then Φ and Φ' are K-satisfiable and from the tree model M of Φ' we can construct an L-model of φ

A Calculus for SNF$_{sml}$

- We do not have an implementation of the calculus for SNF$_{sml}$
- Instead there are two implementations of related calculi
  1. Global Modal Resolution (GMR) calculus:
     - Overapproximates every clause $S : \psi$ by $\mathbb{N} : \psi$, except for $\{0\} : t_\varphi$
     - Generates instances of axioms 'on-the-fly' by additional inference rules
  2. Modal-layered Resolution (MLR) calculus:
     - Allows only singleton labelling sets $\{ml\} : \psi = ml : \psi$
     - Approximate $S : \psi$ by $ml_1 : \psi, \ldots, ml_k : \psi$
       where $ml_i \in S$, $ml_i \leqslant b^L_\varphi$, b^L_φ is a logic- and formula-dependend bound

- As satisfiable benchmark formulae we use 100 S5-satisfiable formulae
  $\leadsto$ satisfiable in any logic of the modal cube
  $\leadsto$ effort to find a model varies depending on logic
- As unsatisfiable benchmark formulae we use 100 K-unsatisfiable formulae that are modified for each logic so that logic specific reasoning is required
  $\leadsto$ effort to find a refutation varies depending on logic



S5-Satisfiable	K-Unsatisfiable
k_poly_n	k_branch_p
s4_md_n	k_path_p
s4_ph_n	k_ph_p
s4_path_n	k_poly_p
s4_s5_n	k_t4p_p

20 formulae in each family (100 for each logic)

K-Unsatisfiable formulae
Replace each propositional variable p by $p \vee \psi_L^p$:

Logic L	ψ_L^p
K	**false**
KB	$(\neg q_p \wedge \Diamond\Box q_p)$
KDB	$(\neg q_p \wedge \Diamond\Box((\Box\neg q_p' \wedge \Box q_p') \vee q_p))$
KTB	$(\neg q_p \wedge \Diamond\Box((\neg q_p' \wedge \Box q_p') \vee q_p))$
KD	$(\Box\neg q_p \wedge \Box q_p)$
KT	$(\neg q_p \wedge \Box q_p)$
K4	$(\Box q_p \wedge \Diamond\Diamond\neg q_p)$
K4B	$(\neg q_p \wedge \Diamond\Diamond\Box q_p)$
KD4	$(\Box q_p \wedge \Diamond\Diamond\Box\Diamond\neg q_p)$
K5	$(\Diamond\neg q_p \wedge \Diamond\Box q_p)$
KD5	$((\Box\neg q_p \wedge \Box q_p) \vee (\Diamond\Box q_p' \wedge \Diamond\neg q_p')$
K45	$(\Box q_p \wedge \Diamond\Box q_p' \wedge \Diamond\Diamond(\neg q_p \vee \neg q_p'))$
KD45	$((\Box\neg q_p' \wedge \Box q_p') \wedge$ $(\Box q_p \wedge \Diamond\Box q_p' \wedge \Diamond\Diamond(\neg q_p \vee \neg q_p'))$
S4	$(\neg q_p' \wedge \Box(\neg q_p' \vee \Box q_p) \wedge \Diamond\Diamond\neg q_p)$
S5	$((\neg q_p \wedge \Box q_p) \vee (\neg q_p' \wedge \Diamond\Diamond\Diamond\Box q_p')$

- Provers started to implement modal logic specific simplifications, e.g.,

$$\mathsf{K4} : \Diamond\Diamond\psi \Longrightarrow \Diamond\psi$$

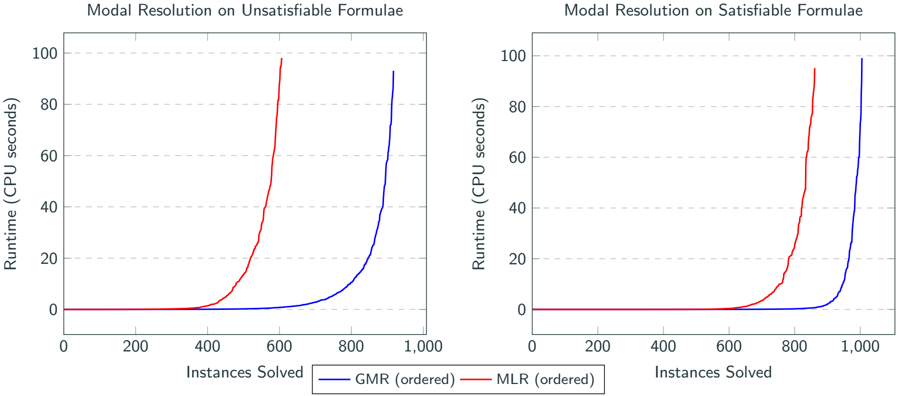
$\leadsto$ allows such provers to undo the replacement of p by $p \vee \psi^p_L$

- The following alternative modifications mostly disable those modal logic specific simplifications

Logic L	ψ^p_l
K	**false**
KB	$((\neg q^0_p \vee \neg q^1_p) \wedge \Diamond\Box q^0_p \wedge \Diamond\Box q^1_p)$
KDB	$((\Box(\neg q^0_p \wedge \neg q^1_p) \wedge \Box(q^0_p \vee q^1_p))$ $\vee ((\neg q^0_p \vee \neg q^1_p) \wedge \Diamond\Box q^0_p \wedge \Diamond\Box q^1_p))$
K4	$(\Box(q^0_p \vee q^1_p) \wedge \Diamond\Diamond(\neg q^0_p \wedge \neg q^1_p))$
K5	$(\Diamond\neg q^0_p \wedge \Diamond\Box q^0_p)$
KB4	$((q^0_p \vee \neg q^1_p) \wedge \Diamond(q^2_p \wedge \Diamond\Box q^0_p)$ $\wedge \Diamond(\neg q^2_p \wedge \Diamond\Box q^1_p))$
K45	$(\Box q^0_p \wedge \Diamond\Box q^1_p \wedge \Diamond\neg q^2_p$ $\wedge \Diamond(q^2_p \wedge \Diamond(\neg q^0_p \vee \neg q^1_p)))$
KD	$(\Box(\neg q^0_p \wedge \neg q^1_p) \wedge \Box(q^0_p \vee q^1_p))$
KT	$(\neg q^0_p \wedge \neg q^1_p \wedge \Box(q^0_p \vee q^1_p))$

Logic L	ψ^p_l
KTB	$((\neg q^0_p \wedge \neg q^1_p \wedge \Box(q^0_p \vee q^1_p))$ $\vee ((\neg q^0_p \vee \neg q^1_p) \wedge \Diamond\Box q^0_p \wedge \Diamond\Box q^1_p))$
KD4	$((\Box(\neg q^0_p \wedge \neg q^1_p) \wedge \Box(q^0_p \vee q^1_p))$ $\vee (\Box(q^0_p \vee q^1_p) \wedge \Diamond\Diamond(\neg q^0_p \wedge \neg q^1_p)))$
S4	$(q^0_p \wedge \Box(\neg q^0_p \vee \Box q^1_p) \wedge \Diamond(q^2_p \wedge \Diamond(\neg q^1_p \wedge \neg q^2_p)))$
KD5	$((\Box(\neg q^0_p \wedge \neg q^1_p) \wedge \Box(q^0_p \vee q^1_p))$ $\vee (\Diamond\neg q^0_p \wedge \Diamond(q^0_p \wedge \Box q^0_p)))$
S5	$((\neg q^0_p \wedge \neg q^1_p \wedge \Box(q^0_p \vee q^1_p))$ $\vee (\neg q^1_p \wedge \Diamond((q^2_p \wedge \neg q^3_p) \wedge \Diamond((q^3_p \wedge \neg q^4_p) \wedge \Diamond(q^4_p \wedge \Box q^1_p)))))$
KD45	$((\Box(\neg q^0_p \wedge \neg q^1_p) \wedge \Box(q^0_p \vee q^1_p))$ $\vee (\Box q^0_p \wedge \Diamond(q^0_p \wedge \Box q^1_p) \wedge \Diamond\neg q^2_p \wedge \Diamond(q^2_p \wedge \Diamond(\neg q^0_p \vee \neg q^1_p))))$

How well does Idea 3 work?



Conclusions, Future Work, Questions

- Global Modal Resolution currently offers the best overall performance on the logics of the modal cube
- The normal form used for Global Modal Resolution is closely related to the definitional clause normal form which performed worse
- The performance gain is therefore most likely linked to
  - the hyperresolution-like inference rules of Global Modal Resolution
  - the use of on-the-fly generated instances of axioms instead of relational frame properties

  Future work: Extend the comparison to include the axiomatic translation from modal to first-order logic to disentangle these factors

- Our modal resolution approach offers a number of advantages:
  - provides decision procedures for all 15 logics of the modal cube
  - provides proofs
  - provides models (implemented only for the ordered refinement)
- But there are faster provers for specific logics
- In particular, the fastest prover for the six logics K, KB, KD, KT, K4, K5 offers neither proofs nor models
- We have also seen that the relational translation without the use of definitional clause normal form can result in better performance though it does not provide a decision procedure
- Question: What criteria should we use for the inclusion or exclusion of an approach / prover in a comparison?